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TURBULENT BOUNDARY LAYER OF A DISSOCIATED GAS AT THE INLET SECTION OF A TUBE

A.N. Leont'ev, B.P. Mironov and A.V. Fafurin

ABSTRACT: A previous solution to the development of a turbulent boundary layer of a gas under nonisothermal conditions has been extended to include the flow of a high-enthalpy gas (in which case it is necessary to consider the effect of the dissociation of the gas on the characteristics of the turbulent boundary layer). The analysis is developed for a mixture of several gases in frozen state inside the boundary layer and in equilibrium state at its boundaries. Formulas are derived for the friction and heat-transfer laws. A solution is obtained to the turbulent boundary-layer equations for the inlet section of the tube on the assumption of a constant wall temperature and subsonic velocities of the gas flow.

A number of theoretical and experimental papers [1-4] have been devoted to the study of the turbulent boundary layer in the inlet section of the channel. /100*

However in the majority of studies it is the flow of an incompressible liquid with constant physical parameters that has been investigated. For many practical problems it would be of interest to analyze the development of a turbulent boundary layer of a gas in the inlet section of a tube under the conditions of intrinsic nonisothermicity. A solution to such a problem, based on the use of the theory of boundary laws, is given in [1]. In the present article this solution is extended to the case of a high-enthalpy flow of gas, when the influence of dissociation of the gas on the characteristics of the turbulent boundary layer must be taken into account. We analyze the flow of a mixture of i gases, which is found in "frozen" state inside the boundary layer and in equilibrium state at its boundaries. Formulas are derived for the laws of friction and heat-transfer and a solution is given to the equations of a turbulent boundary layer for the inlet section of the tube at a constant wall temperature in the region of subsonic flow rates of the gas.

* Numbers in the margin indicate pagination in the foreign text.

1. Relative Law of Friction for a Turbulent Boundary Layer of a Dissociated Gas.

According to [1], the relative coefficient of turbulent friction is defined by the expression

$$\Psi=\left[\frac{1}{Z}\int_{w_1}^{1}\left(\frac{\rho}{\rho_0}\frac{\tau_0^*}{\tau^*}\right)^{1/2}d\omega\right]^2,\quad Z=\left(\frac{c_{f_0}}{2}\right)^{1/2}\int_{\xi_1}^{l}\left(\frac{\tau_0^*}{1-\beta}\right)^{1/2}\frac{d\xi}{\bar{l}}$$
$$\Psi=\left(\frac{c_f}{c_{f_0}}\right)_{R^{++}},\qquad c_f=\frac{2\tau_w}{\rho_0 w_0^2},\qquad c_{f_0}=\frac{2\tau_{w0}}{\rho_0 w_0^2} \tag{1.1}$$

Here Ψ is the ratio of the coefficient of friction under the conditions analyzed to the coefficient of friction under standard conditions with identical values of R^{++}.

From the equations

$$p=\rho RT,\qquad h_0=\sum_i \alpha_{i0}h_{i0},\qquad h_i=\int_0^T c_{p_i}dT+h_i^\circ \tag{1.2}$$

we can obtain the relationship between the density of the gas and the enthalpy.

The degree of dissociation of air for a low-temperature plasma (T = 5000° K) is on the order of 0.3. For this region the parameter μc_p must be assumed constant in cross section of the boundary layer. Then

$$\frac{\rho}{\rho_0}=\frac{h_0-\Sigma\alpha_{i0}h_i^\circ}{h-\Sigma\alpha_i h_i^\circ} \tag{1.3}$$

Here and in the following, summation is carried out over the index i. By substituting (1.2) into (1.3) and assuming that the fields are similar in velocity, enthalpy and concentration, [2], we obtain

$$\frac{\rho}{\rho_0}=\frac{1-\Sigma\alpha_{i0}\psi_i}{\psi_h-\Sigma\alpha_{wi}\psi_i+\{1-\psi_h-\Sigma(\alpha_0-\alpha_w)_i\psi_i\}\omega},\qquad \left(\psi_h=\frac{h_w}{h_0},\ \psi_i=\frac{h_i^\circ}{h_0}\right) \tag{1.4}$$

In Equations (1.1) - (1.4) we have taken the definitions: p represents the pressure, ρ the density, h the enthalpy, h_i° the heat of formation of the i-th component, d the degree of dissociation, $\bar{l}$ the relative path length of the mixing, ω the dimensionless velocity, T the temperature, R the gas constant; and the indexes w represent the parameters on the wall and 0 those in the main body of the stream.

Equation (1.4) establishes the relationship between the density and the velocity of the gas at a given point with specified parameters of the gas at the boundaries of the boundary layer. Strictly speaking, Equation (1.4) is precise only for a gradient-free gas flow, with a constant value of the enthalpy on the wall with a Prandtl number P_T = 1.0.

However, as calculations show, this equation can be used for all practical purposes for more complex boundary conditions as well, in particular for the flow of a gas at the inlet section of the tube.

For computing the parameter Z let us make the assumption that is ordinarily used in semi-empirical theories of turbulence: $\beta = 0$, /101 $l = \kappa\xi$ and $\tau_0^* = 1.0$, then

$$Z = -\sqrt{1/2 c_{f_0}}\,\varkappa^{-1} \ln \xi_1 \qquad (1.5)$$

From analysis of the experimental data on velocity distribution by cross section of the turbulent boundary layer with intrinsic nonisothermicity [4-6], it follows that the dimensionless velocity at the boundary of a viscous sub-layer and the relative thickness of the viscous sub-layer retain the same values as for an incompressible liquid if the physical parameters of the gas are defined by the wall temperature.

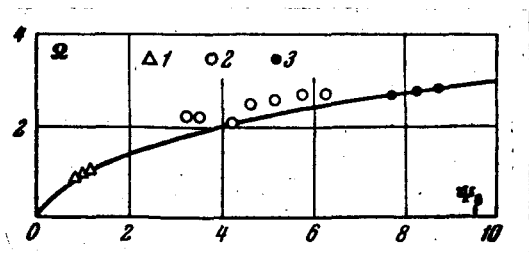

Fig. 1. Effect of Nonisothermicity on the Parameters of a Viscous Sub-Layer. The Curve is Calculated from Formula (1.7); Points 1, 2, 3 Represent Experiments from References [4-6].

$$\frac{w_1}{w_0\sqrt{1/2 c_f \psi_\rho}} = \frac{\delta_1 w_0 \sqrt{1/2 c_f \psi_\rho}}{\nu_w} = 11.6 \qquad (1.6)$$

Hence

$$\omega_1 = 11.6\sqrt{1/2 c_{f_0} \Psi \psi_\rho},\quad \xi_1 = \frac{135\psi_\rho \delta^{++}}{\omega_1 R^{++}\delta} \qquad (1.7)$$

$$R^{++} = \frac{\rho_0 w_0 \delta^{++}}{\mu_w},\quad \psi_\rho = \frac{\rho_0}{\rho_w} \qquad (1.8)$$

Comparison of Formula (1.7) with the experiments of various authors [4-6] is shown on Figure 1, where $\Omega = w_1/11.6 \cdot w_0 \sqrt{1/2 c_f}$.

By substituting (1.7) into (1.5) we have

$$Z = -\frac{\sqrt{1/2 c_{f0}}}{\varkappa} \ln \frac{135 \psi_\rho \delta^{++}}{\omega_1 R^{++} \delta} \tag{1.9}$$

The coefficient of friction c_{f0} under standard conditions is defined by the Karman formula

$$1/2 c_{f0} = (2.5 \ln R^{++} + 3.8)^{-2} \tag{1.10}$$

As shown in Reference [7,8], the Reynolds number R^{++} which appears in Formula (1.10), should be determined from (1.8).

For the standard conditions we have [1]

$$Z = 1 - \omega_{10} \tag{1.11}$$

We may expect that such a simple relationship between Z and ω_1 will also be retained for complex conditions. In fact, comparison of the results of the calculation (Fig. 2) from Formula (1.9) with those from Formula (1.11) and using for the velocity distribution the rule of "one seventh", the limiting dependence of the function Ψ and (1.3) for different values of ψ_ρ and R^{++}, shows that for the practical region of variation in ψ_ρ and R^{++} we can use the formula

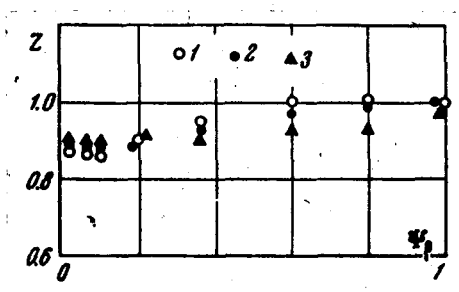

Fig. 2. Effect of Nonisothermicity on the Parameter $Z = z/(1-\omega_1)$. Computations of Z and ω_1 are made from Formulas (1.9) and (1.7); in this case Point 1 represents $R^{++} = 500$; Point 2, $R^{++} = 10^4$; Point 3, $R^{++} = 10^8$.

$$Z = 1 - 11.6 \sqrt{1/2 c_{f0} \Psi \psi_\rho} \tag{1.12}$$

By substituting Equations (1.4), (1.7) and (1.12) into Equation (1.1) and assuming that (1.9) will be valid during dissociation, after integration we find the relative law of friction of a turbulent boundary layer for the conditions analyzed

$$\psi_h = \frac{(1 - \Sigma \alpha_{i0} \psi_i)(2 - \sqrt{\Psi})^2 + \Psi \Sigma \alpha_{wi} \psi_i - [1 - \Sigma (\alpha_0 - \alpha_w)_i \psi_i] 8.2 \sqrt{c_{f0} \psi_\rho \Psi} \Psi}{\Psi (1 - 8.2 \sqrt{c_{f0} \psi_\rho \Psi})} \tag{1.13}$$

If $\alpha_0 = \alpha_w = 0$, then

$$\psi_h = 1 + \frac{4(1 - \sqrt{\Psi})}{\Psi (1 - 8.2 \sqrt{c_{f0} \Psi \psi_h})} \tag{1.14}$$

When $R^{++} \to \infty$, $Z \to 1$, $\omega_1 \to 0$, from Equation (1.13) it also follows that /102

$$\Psi_\infty = 4 \left[1 + \left(\frac{\psi_h - \Sigma \alpha_{wi} \psi_i}{1 - \Sigma \alpha_{0i} \psi_i} \right)^{1/2} \right]^{-2} \tag{1.15}$$

For an undissociated gas we obtain the familiar result [1]

$$\Psi_\infty = 4(\sqrt{\psi_h}+1)^{-2} \tag{1.16}$$

The maximal divergence between Formulas (1.14) and (1.16), for $R^{++} = 300$, $\psi_h \approx 0.1$, is about 25%. Thus, for practical computations we can use the limiting relative law of friction in the form of (1.15).

Figure 3 shows a comparison of the results of computation using Formula (1.13) with the experiments of N.M. Belyanin [3]. We can see a satisfactory agreement between the proposed theory and the experiment.

§2. Development of a Dynamic Turbulent Boundary Layer on the Inlet Section of a Tube.

The integral equation of impulses for the inlet section of a tube can be represented in the form [1]

$$\frac{dR^{++}}{dX} + \frac{R^{++}}{W_0}\frac{dW_0}{dX}(1+H) = R_{D1}\Psi\frac{c_{f0}}{2} \tag{2.1}$$

Here $W_0 = w_0/w_{01}$ is the relative velocity in the unperturbed main body of the stream, W_{01} is the velocity at the inlet into the tube.

Equation (2.1) includes the parameter H, which is equal to the ratio of the depth of displacement to the depth of impulse loss. Let us analyze the influence of nonisothermicity and dissociation on the size of this parameter. The physical depth of displacement δ^+ is related to the depth of displacement in Dorodnitsin variables δ_1^+ by the equation

$$\delta^+ = \delta_1^+ - \int_0^\delta \left(\frac{\rho}{\rho_0} - 1\right)\left(1 - \frac{y}{R_0}\right)dy \tag{2.2}$$

If we take into account Equation (1.4) we have

$$\delta^+ = \delta_1^+ - \int_0^\delta \frac{(h - \Sigma\alpha_i h_i^\circ) - (h_0 - \Sigma\alpha_{i0}h_i^\circ)}{h_0 - \Sigma\alpha_{i0}h_i^\circ}\left(1 - \frac{y}{R_0}\right)dy \tag{2.3}$$

$$\delta^+ = \delta_1^+\left\{1 - \frac{1}{1-\Sigma\alpha_{i0}\psi_i}\left[1 - \psi_h - \sum_i(\alpha_0 - \alpha_w)_i\psi_i\right]\right\} \tag{2.4}$$

Whence

$$H = H_1\psi_\alpha, \qquad \psi_\alpha = \frac{\psi_h - \Sigma\alpha_{wi}\psi_i}{1 - \Sigma\alpha_{0i}\psi_i} \tag{2.5}$$

The form parameter H depends only slightly on the nonisothermicity and may be assumed as a constant equal to $H_{10} \approx 1.347$.

Figure 4 shows the results of computing the parameters of an

axially symmetric turbulent boundary layer under the assumption of a wall profile of the velocities (n = 1/7) and by using Formula (1.4) for the density.

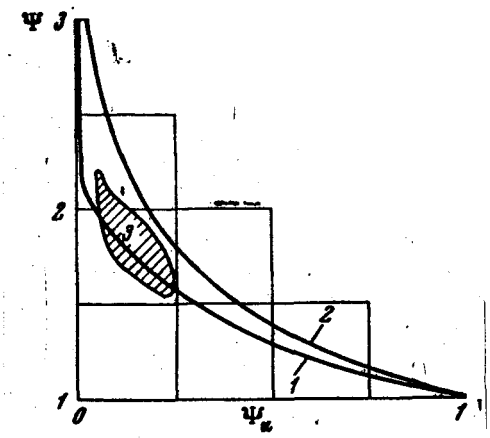

Fig. 3. Comparison of the Computations Using Formula (1.13) With the Experiments of N.M.Belyanin; Computations made from Formula (1.13); Curve 1 is for R^{++} = 300, Curve 2 is for R^{++} = ∞, Area 3 indicates the region of Belyanin's Experiments.

It is clear from the graph that the parameters δ_1^+ and δ^{++} depend substantially on the nonisothermicity, however the size of H_1 remains practically constant and equal to 1.347. Figure 5 shows the comparison of the results of computing the form parameter H with Formula (2.5) under the same assumptions. The influence of the dissociation of air on the size of the form parameter H is clearly obvious.

For the absence of dissociation, we obtain $H = H_1\psi h$ from Equation (2.5)

Thus, for the boundary conditions ψ_α = const the form parameter H may be assumed constant along the length of the tube.

To compute the turbulent boundary layer with an external streamline flow of bodies, it is sufficient to use Equations (2.1), (1.15) and (1.10), since the velocity on the external boundary of the boundary layer W_0 is determined from the conditions of potential streamline flow of a body and is a known function of the longitudinal coordinate. During the flow of a gas at the inlet section of a tube the velocity in the main body of the stream is the sought function, and to determine it we must use the equation of discontinuity. The equation of discontinuity for a circular tube can be written conveniently in the form [1] /103

$$\rho_{01}W_{01} = \rho_0 W_0 (1 - 2\delta^+ / R_0) \qquad (2.6)$$

By introducing the form parameter H, we have

$$R^{++} = {}^1/_4 R_{D1} (W_0 - 1) / H \qquad (2.7)$$

The system of equations (2.1), (1.15), (1.10) and (2.7) is a closed one, and as a result of its solution we find the desired relationship between the parameters W_0, R^{++}, $\frac{1}{2}c_f$ and the length of the tube x/D.

The law of friction for the standard conditions (1.10) can be

conveniently approximated by a power function such as

$$c_{f0} = B / (R^{++})^m \qquad (2.8)$$

Here the values of the coefficients B and m depend on the range of variation in R^{++}.

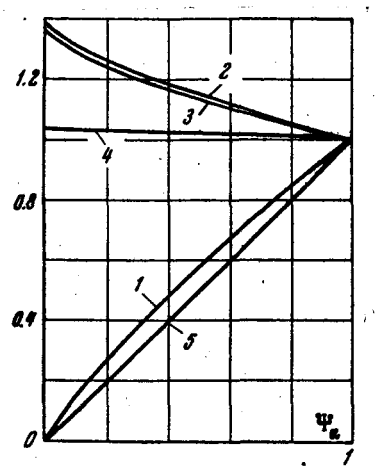

Fig. 4

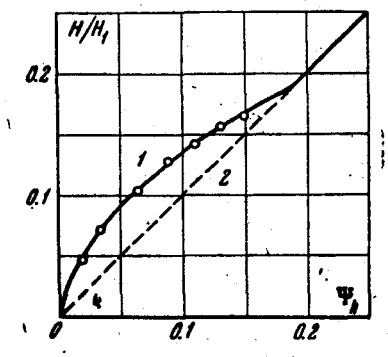

Fig. 5

Fig. 4. Effect of the Nonisothermicity on the Parameters of an Axially Symmetric Turbulent Boundary Layer. *1* - $\delta^+/\delta^{+\circ}$, *2* - $\delta_1^{++}/\delta_1^{+\circ}$, *3* - $\delta^{++}/\delta^{++\circ}$, *4* - $H_1/H_{1\text{var}}$, *5* - δ^+/δ^{++}

Fig. 5. Effect of the Dissociation of Air on the Size of the Form Parameter: *1* - Calculation using Equation (2.5); *2* - Without Allowing for the Dissociation; The Points Represent Computations - Made By Integral Parameters. All Computations have been made for T_w = 300° K.

For R^{++} from 300 to 10^4 we may assume 1/2 B ▫ 0.0128 and m=0.25.

In this case we are able to obtain an analytical solution to the original system of equations for the conditions h_w = const. Taking into account equations (2.7) and (2.8), equation (2.1) can be written in the form

$$\left[\frac{(W_0-1)^m}{W_0}+\frac{(W_0-1)^{m+1}}{W_0^2}(1+H)\right]\frac{dW_0}{dX}=\Psi\frac{B}{2}\frac{(4H)^{1+m}}{(R_{D1})^m} \qquad (2.9)$$

if we assume at the inlet to the tube a uniform profile of the velocities and if we take into account that for the conditions h_w = const, Ψ = const and H = const (the case of subsonic velocities of the gas flow is analyzed), and if we also assume that 1/2 B = 0.0128 and that m = 0.25, from Equation (2.9) we obtain

$$\left[(1+H)\frac{5}{4}+1\right]\left[4(W_0-1)^{0.25}-\frac{1}{\sqrt{2}}\ln\frac{(W_0-1)^{0.5}+\sqrt{2}(W_0-1)^{0.25}+1}{(W_0-1)^{0.5}-\sqrt{2}(W_0-1)^{0.25}+1}-\right.$$
$$\left.-\sqrt{2}\operatorname{arc\,tg}\frac{\sqrt{2}(W_0-1)^{0.25}}{1-(W_0-1)^{0.5}}\right]-(1+H)\frac{(W_0-1)^{1.25}}{W_0}=\Psi\frac{0.0725H^{1.25}}{R_{D1}^{0.25}}X \qquad (2.10)$$

For the region of subsonic velocities of the flow of an undissociated gas, Equation (2.10) will result in Equation (6.35) from Reference [1]; only the viscosity of a gas entering into R_{D1} is determined from the wall temperature.

Figure 6 shows the results of computing W_0 as a function of $\xi = X/R_{D1}{}^m$ using Equation (2.10) for the various degrees of dissociation and nonisothermicity. Equation (2.10) is valid only for the inlet section of the tube as long as the dynamic and thermal boundary layers are not closed.

Let us define the length of the inlet section from the condition that at the end of the section of stabilization the thickness of the boundary layer becomes equal to the radius of the tube. Then /104

$$R_H{}^{++}=R_D\frac{1}{2}\frac{\delta^{++}}{R_0}=R_{D1}\frac{W_{0H}}{2}\frac{\delta^{++}}{R_0} \qquad (2.11)$$

On the other band,

$$R_H{}^{++}=\frac{R_{D1}(W_{0H}-1)}{4H} \qquad (2.12)$$

From Equations (2.11) and (2.12) we find

$$W_{0H}=\frac{1}{1-2\delta^{++}H/R_0} \qquad (2.13)$$

Parameters δ^{++}/R_0 and H in first approximation can be determined from Figure 4. If we substitute the value of W_{0H} into Equation (2.10), we can determine the length of the inlet section

$$X_H=\frac{R_{D1}^{0.25}}{\Psi 0.0725H^{1.25}}\left\{\left[(1+H)\frac{5}{4}+1\right]\left[4(W_{0H}-1)^{0.25}-\right.\right.$$
$$-\frac{1}{\sqrt{2}}\ln\frac{(W_{0H}-1)^{0.5}+\sqrt{2}(W_{0H}-1)^{0.25}+1}{(W_{0H}-1)^{0.5}-\sqrt{2}(W_{0H}-1)^{0.25}+1}- \qquad (2.14)$$
$$\left.\left.-\sqrt{2}\operatorname{arc\,tg}\frac{\sqrt{2}(W_{0H}-1)^{0.25}}{1-(W_{0H}-1)^{0.5}}\right]-(1+H)\frac{(W_{0H}-1)^{1.25}}{W_0}\right\}$$

The broken line on Figure 6, computed from Equations (2.13) and (2.14), defines the length of the inlet section and the region of applicability of the relationships obtained. It should be noted that dissociation of a gas has a considerable effect on the length of the section of stabilization.

The sequence of computing the dynamic turbulent boundary layer

in the inlet section of the tube is the following:

1. The parameters Ψ_∞ and H are determined from the specific parameters α_w, α_0 and h_w from Formulas (1.15) and (2.5).

2. The length of the segment of stabilization X_H is determined from Equations (2.13) and 2.14).

3. W_0 as a function of X is computed from Equation (2.10) or the graph on Figure 6.

4. The values of R^{++} are determined from Equation (2.7) and the values of the coefficients of friction are computed from Equation (2.8) and (1.15) or (1.13).

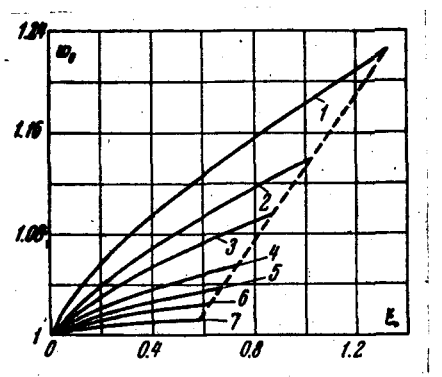

Fig. 6. The Dimensionless Value of the Velocity W in the Main Body of the Stream as a Function of the Length of the Tube $\xi = XR^{-0.25}$ at different values of Ψ; Curves *1*,..,*7* represent the values ψ_α = 1.06, 0.4, 0.2, 0.137, 0.0875 and 0.044. The broken line defines the boundary of the inlet section (Computations made from Formula (2.14).

§3. The Development of a Thermal Turbulent Boundary Layer at the Inlet Section of a Tube.

The integral equation of energy for the inlet section of a tube can be conventiently written in the form [1]

$$\frac{dR_h^{++}}{dX} + \frac{R_h^{++}}{\Delta h}\frac{d(\Delta h)}{dX} = R_{D1}W_0\Psi_\varepsilon S_0 \qquad (3.1)$$

Here S_0 is the Stenton criterion under standard conditions: For the case h_w = const

$$dR_h^{++}/dX = R_{D1}W_0\Psi_\varepsilon S_0 \qquad (3.2)$$

With the assumptions made the law of heat exchange has the same form as the law of friction

$$\Psi_\varepsilon = 4\left[1 + \left(\frac{\psi_h - \Sigma\alpha_{wi}\psi_i}{1 - \Sigma\alpha_{0i}\psi_i}\right)^{1/2}\right]^{-2} \qquad (3.3)$$

Under standard conditions /105

$$S_0 = B/2\,(R_h^{++})^m P^n \qquad (3.4)$$

Let us assume $1/2\ B \approx 0.0128$, $m = 0.25$ and $n = 0.75$. Then the integral of equation (3.2) is equal to

$$R_h^{++} = \left[\frac{(1+m)B}{2P^{0.75}} R_{D1}\Psi_* \int_0^X W_0\,dX\right]^{\frac{1}{1+m}} \qquad (3.5)$$

From equation (2.9), it follows that

$$W_0\,dX = \frac{2R_{D1}^m}{B\Psi(4H)^{1+m}}\left[(W_0-1)^m + \frac{(W_0-1)^{m+1}}{W_0}(1+H)\right]dW_0 \qquad (3.6)$$

Consequently,

$$R_h^{++} = \frac{R_{D1}(W_0-1)}{4HP^{0.8n}}\left\{2+H-\frac{(1+H)\,1.25}{(W_0-1)^{1.25}}\left[4(W_0-1)^{0.25} - \right.\right.$$
$$\left.\left. - \sqrt{2}\,\mathrm{arc\,tg}\,\frac{\sqrt{2}(W_0-1)^{0.25}}{1-(W_0-1)^{0.5}} - \frac{1}{\sqrt{2}}\ln\frac{(W_0-1)^{0.5}+\sqrt{2}(W_0-1)^{0.25}+1}{(W_0-1)^{0.5}-\sqrt{2}(W_0-1)^{0.25}+1}\right]\right\}^{0.8} \qquad (3.7)$$

If we know the dependence of W_0 on X, which is determined by Equation (2.10), we can find the local values of the criterion Rh^{++} from Equation (3.7) and determine the local value of the Stenton criterion S from Formulas (3.4) and (3.3).

From Equations (2.7), (2.8), (3.4) and (3.7) we can find the relationship between the local values of the coefficients of friction and heat transfer for the conditions analyzed

$$\frac{c_f}{2S} = P^{0.75}\left\{2+H-\frac{(1+H)\,1,25}{(W_0-1)^{1.25}}\left[4(W_0-1)^{0.25} - \right.\right.$$
$$\left.\left. - \sqrt{2}\,\mathrm{arc\,tg}\,\frac{\sqrt{2}(W_0-1)^{0.25}}{1-(W_0-1)^{0.5}} - \frac{1}{\sqrt{2}}\ln\frac{(W_0-1)^{0.5}+\sqrt{2}(W_0-1)^{0.25}+1}{(W_0-1)^{0.5}-\sqrt{2}(W_0-1)^{0.25}+1}\right]\right\}^{0.2} \qquad (3.8)$$

The computations carried out using Equation (3.8) show that the gradient of pressure in the inlet section has no substantial effect on the Reynolds analogy.

The proposed method of computations can be extended to the case of a random distribution h_w along the length of the channel for specific sections with a constant enthalpy of the gas on the wall.

In conclusion the authors wish to express their appreciation to S.S. Kutateladze for his discussions of the project.

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